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Heisenberg Uncertainity Principle :-

In Classical mech., particle's position, mom., energy etc. can be calculated simultaneously bc to C.M. particle behave like particle.

Acc. to Q. Mech., If we know the position but the corresponding momentum will be uncertain. So in Q.M. we can not calculate the position & mom. simultaneously with accuracy

$$\boxed{\Delta x \, \Delta p_x \geq \frac{\hbar}{2}}$$

$$\Delta x \cdot \Delta K \geq \frac{\hbar}{2}$$

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$$

For Canonically conjugate ~~classical pair~~ dynamical vari pair (the unit of the product of position & mom will be unit of action)

& pair which satisfies the Canonical eqn of motion $\dot{q} = \frac{\partial H}{\partial p}$ & $\dot{p} = \frac{-\partial H}{\partial q}$

will be conjugate variable. Uncertainity principle is valid only for Conjugate dynamical var.

Position space wave funcn & mom. sp wave funcn are fourier transform of each other i.e. we can convert position space wave fun into mom. space wave funcn or vice versa.

If two operator commute then we get ~~uncertainity~~ soln. Uncertainity product = 0 i.e. we can measure pos mom. simultaneously

$$[A, B] = 0 \Rightarrow AB = BA$$

$$[x, p_x] = 0$$

$$\Delta x \cdot \Delta p_x \geq \hbar/2$$

$$\Delta x \cdot \Delta p = 0$$

Applications

(1) Non Existance of e^- inside the Nucleus :-
(2) To calculate the Bohr radius
(3) To calculate the ground state energy

(1) Non Existance of e^- inside the nucleus :-

Suppose e^- exist inside the nucleus then uncertai will be of the order of size of nucleus
so uncertainity in position

$$\Delta r = 2 \times 10^{-15} \text{ (Radius of nucleus)}$$

$$\Delta r . \Delta p \geq \frac{\hbar}{2}$$

$$\Delta p \geq \frac{\hbar}{2 \Delta r}$$

$P \rightarrow P - \Delta P$ to $P + \Delta P$

$$p \sim \Delta p$$

$$K.E. = \frac{p^2}{2m} = \frac{(\Delta p)^2}{2m} = \frac{1}{2m}\left(\frac{\hbar}{2\Delta r}\right)^2 = 97 \text{ or } 96 \text{ MeV}$$

If e^- exist in the nucleus then K.E. of e^- is 97 or 96 MeV acc. to Uncertainity principle
But the experimental value of K.E. of e^- is 4 MeV.

There is a large difference in K.E. So e^- can not exist inside the nucleus.

(II) & (III) Bohr radius & ground state ~~properties~~ Energy :-

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

Min uncertainity $\Delta x \, \Delta p = \frac{\hbar}{2}$

$$\text{total energy} = \frac{p^2}{2m} + V$$

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$p \to \Delta p$ If V is func^n of x or (r in 3 dim) than
$x \sim \Delta x$

hence energy will be the func^n of Δx,

$$E(\Delta x) = \frac{p^2}{2m} + V$$

$$\frac{dE}{d(\Delta x)} = 0$$

$$E(\Delta x) = \frac{(\Delta p)^2}{2m} + V(x) \qquad \Delta x \cdot \Delta p = \frac{\hbar}{2}$$

$$= \frac{(2\Delta x)^2}{2m} + V(\Delta x) \qquad \Rightarrow \Delta p = \frac{\hbar}{2\Delta x}$$

$$E = \frac{2(\Delta x)^2}{m} + V(\Delta x)$$

$$\frac{dE}{d(\Delta x)} = 0 \Rightarrow \frac{2}{m} \cdot 2\Delta x \cancel{=0} + \frac{dV}{d(\Delta x)} = 0$$